

A methodology for the constrained optimization of tunnels with GiD

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Abstract

In this paper we merely want to explore the possibilities of GiD as a powerful interface to deal with complex optimization problems. A general methodology is developed to incorporate GiD as an external aid-software. The proposed methodology has two main steps. First the problem statement must be built expressing explicit relationships between design variables and objective functions and constraints. In order to do that, for whatever problem you arise, we use interpolation schemes over a collection of different designs. The second step is the minimization of the problem using any relevant technique you can apply. Both steps can be repeated in an iterative process to find a minimum. The optimization of tunnels is a good example to evaluate the possibilities of this methodology. Moreover it can be extended to other types of problems like fluids, thermal, etc.

1 Introduction

The goal of the optimization of structures is to find values for some design variables:

$$\mathbf{x} = \{x_1, x_2, \dots, x_n\} \quad (1)$$

Minimizing an objective function:

$$f(\mathbf{x}) = f(x_1, x_2, \dots, x_n) \quad (2)$$

And satisfying, at the same time, some design constraints:

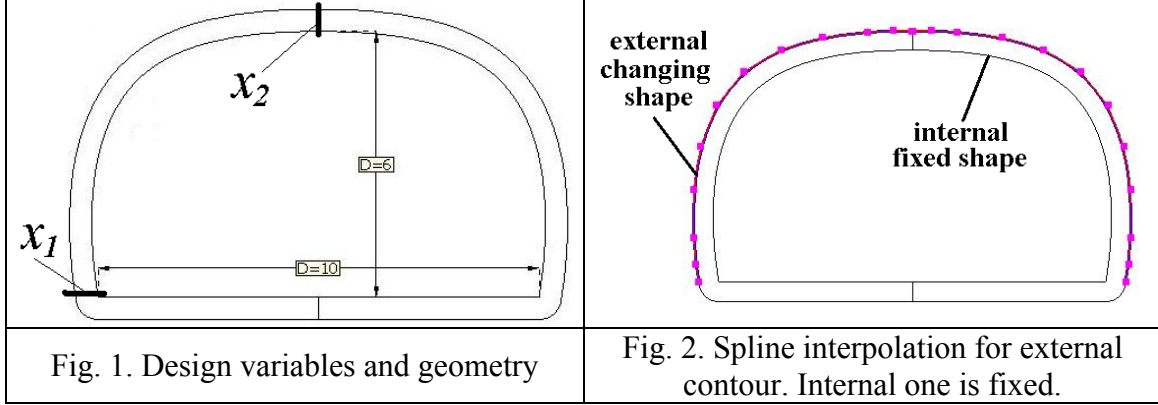
$$g(\mathbf{x}) = g_i(x_1, x_2, \dots, x_n) = 0 \quad i = 1 \dots m \quad (3)$$

Constraints are almost referred to a maximum allowable stresses or displacements according to normative and material capabilities. It is usual to express constraints as inequalities; nevertheless all inequalities can be converted in expressions like (3) with the help of slack variables [1]. Normally constraints are evaluated using the solution of the boundary problem of the structure. The equilibrium equation can be written as [2], [3]:

$$\int_V \delta \boldsymbol{\varepsilon} \boldsymbol{\sigma} dV = \int_V \delta \mathbf{u} \mathbf{b} dV + \int_{\partial V} \delta \mathbf{u} \mathbf{t} dS \quad (4)$$

Where $\delta \epsilon$ refers a virtual deformation, σ internal stresses of the structure, δu a field of virtual displacements and b, t external loads over volume and surface. Finite Elements Method FEM is a common technique that solves equation (4).

In this paper we want to optimize the inner reinforcement of a tunnel with an internal shell of concrete and steel. In particular, we want to optimize the volume of concrete and we have constraints in stresses and in the geometrical shape definition. This problem is complex but we simplify it avoiding some anchoring problems [4], [5] and material anisotropies [6]. To increase complexity neither objective function nor constraints depend in explicit manner from the design variables in this problem.



2 Methodology

The methodology has two main steps. The first one builds an analytical expression of the problem and the second one minimizes the problem.

2.1 First step: Building the problem

Design variables are the base thickness x_1 and the crown x_2 . The objective function is the area of concrete. This area A must be minima. Area depends on crown and thickness, however it is difficult to write an analytical expression because the external boundary was defined using splines with GiD. With this definition when these design variables change a new design is achieved with GiD creating a homothetic curve. Hence, objective function will be interpolated for simplicity:

$$f(x_1, x_2) = A(x_1, x_2) \quad (5)$$

In this problem we define three constraints: Maximum compression stresses in concrete, maximum displacement in the crown and minimum value for thickness. Neither stresses σ nor δ displacements depend explicitly on design variables, thus an interpolation scheme must also be done for them:

$$g_1(x_1, x_2) = \sigma(x_1, x_2) - \sigma_{\max} \leq 0 \quad (6)$$

$$g_2(x_1, x_2) = \delta(x_1, x_2) - \delta_{\max} \leq 0 \quad (7)$$

The last constraint is a minimum requirement for the base thickness, thus:

$$g_3(x_1) = x_1 - x_1^{\min} \leq 0 \quad (8)$$

Where $\sigma_{\max} = 1500 \text{ N/cm}^2$, $\delta_{\max} = 1 \text{ cm}$ and $x_1^{\min} = 0.05 \text{ m}$.

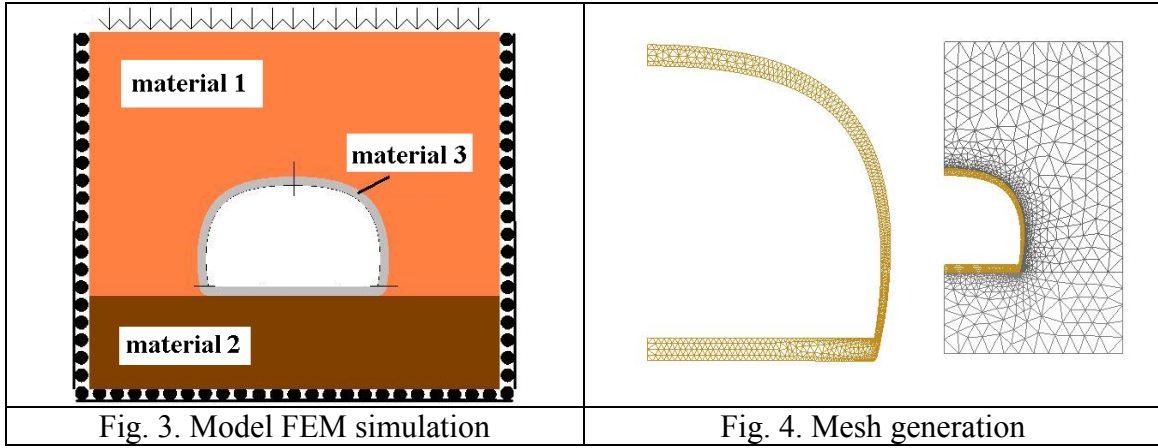
Then we interpolate [7] three quadratic polynomials for the definition of objective function (9) and constraints in stresses (10) and displacements (11):

$$A(x_1, x_2) = a_1 x_1^2 + a_2 x_2^2 + a_3 x_1 + a_4 x_2 + a_5 x_1 x_2 + a_6 \quad (9)$$

$$\sigma(x_1, x_2) = b_1 x_1^2 + b_2 x_2^2 + b_3 x_1 + b_4 x_2 + b_5 x_1 x_2 + b_6 \quad (10)$$

$$\delta(x_1, x_2) = c_1 x_1^2 + c_2 x_2^2 + c_3 x_1 + c_4 x_2 + c_5 x_1 x_2 + c_6 \quad (11)$$

Every interpolation has 6 unknowns a_i, b_i, c_i and according to the strategy six different problems must be defined and solved.



In this case we use the commercial software GiD [8], a mesh generator and post-processor, and the FEM analysis code Calsef [9]. The simulation computes a plane strain problem. Soil and concrete properties are in table 1. There was also a load on the top of 100 KN/m (see figure 3).

	Material 1: upper soil	Material 2: bottom soil	Material 3: concrete
Density	21 KN/m ²	23 KN/m ²	25 KN/m ²
Young modulus	3.0 10 ⁴ KN/m ²	3.0 10 ⁶ KN/m ²	30.0 10 ⁶ KN/m ²
Poisson coefficient	0.35	0.35	0.25

From the results of programs we built data table 2.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Thickness x_1 (m)	0.1	0.25	0.25	0.5	0.5	0.7
Crown x_2 (m)	0.7	0.25	0.5	0.5	0.5	0.7
Area A (m ²)	4.018	2.991	3.95	4.184	5.17	4.44
Stress σ (MPa)	13.61	8.86	7.71	8.49	6.71	10.48
Displacement δ (cm)	0.74	1.323	0.851	1.103	0.712	1.408

Next we found the constants a_i, b_i, c_i from three system of 6 equations (12), (13) and (14). We solved each system of equations with a simple sheet of Excel ®:

$$\begin{bmatrix} A_1 \\ \vdots \\ A_6 \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} a_1 \\ \vdots \\ a_6 \end{bmatrix} \quad \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_6 \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} b_1 \\ \vdots \\ b_6 \end{bmatrix} \quad \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_6 \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} c_1 \\ \vdots \\ c_6 \end{bmatrix} \quad (12), (13) \text{ and } (14)$$

Notice that the system matrix \mathbf{M} (15) is always the same for each interpolation function. This reduces very much the computational effort. However, if you employ different interpolations for objective function and constraints, the system matrix would be different for each one of them.

$$[\mathbf{M}] = \begin{bmatrix} x_{11}^2 & x_{21}^2 & x_{11} & x_{21} & x_{11}x_{21} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{16}^2 & x_{26}^2 & \cdots & \cdots & x_{16}x_{26} & 1 \end{bmatrix} \quad (15)$$

2.2 Second step: Minimization strategy

Once we established the problem we started the minimization. In this case the selected technique was penalization [10], [11]. We used Matlab® and wrote a short minimization code to optimize the structure. Penalization converted the constrained problem to an unconstrained one adding the penalized constraints in the objective function, using equations (5)-(8) we built:

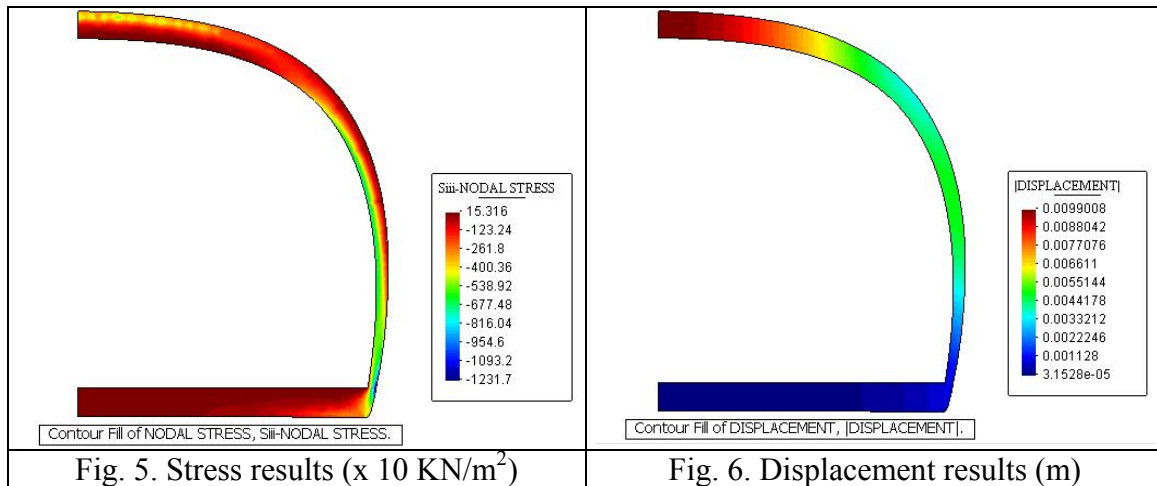
$$f(x_1, x_2) = A(x_1, x_2) + \alpha_1 g_1(x_1, x_2) + \alpha_2 g_2(x_1, x_2) + \alpha_3 g_3(x_1) \quad (16)$$

Penalization technique has the advantage of flexibility because the modification of penalty parameters can help to reach the minimum. The values of penalty parameter $\alpha_1, \alpha_2, \alpha_3$ ranges from 0.01 to 10000.

2.3 Iterative procedure

After solving the minimization problem one solution was found. This solution replaced the worse of the first six designs. Then a new problem statement was defined incorporating this new design and it was minimized again. This iterative technique ended when the solution verified constraints and the improvement in the original objective function (without penalty) was less than 5%. After two trials the final solution was reached, see table 3.

Table 3. Values of the optimum for each iteration		
	Thickness x_1 (m)	Crown x_2 (m)
First iteration	0.05	0.602
Second iteration	0.146	0.47



4 Conclusions

This paper presented a robust and flexible approach to deal with optimization problems for tunnel reinforcement. The proposed methodology takes advantage of GiD and Calsef commercial software. Other common tools like Excel ® and MatLab ® were also incorporated in the optimization process on demand.

The proposed formulation performs well when the objective function or some of the constraints are difficult to write in an explicit manner with respect the design variables. These functions can be interpolated from some desired base of functions. In the example of this paper second order polynomials were used to interpolate objectives and constraints. Nevertheless other interpolation schemes like trigonometric functions, splines, exponential functions, etc. could also be used. The choice for the interpolation basically depends on the nature of the problem, thus unfortunately no universal rule can be defined.

Another advantage of the methodology is that minimization technique is also a free choice. In this example penalty functions performed well, however another technique could be used. Notice that penalty functions does not force the accomplishment of the constraint, only penalizes, thus a quasi-solution near of the real minima can be easy to find. The possibility of adjusting the penalty coefficients gives the chance to manipulate the importance of some particular constraints, relaxing or forcing one or other. This manipulation facility could be considered an inconvenience, because of the sensitivity to the penalization parameters, but on the contrary it must be faced as an advantage. From this point of view, penalty technique is more flexible than other techniques.

We expect that the proposed methodology can be applied to other range of engineering problems like thermal, fluids, mechanics, etc.

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